INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY: APPLIED BUSINESS AND EDUCATION RESEARCH
2026, Vol. 7, No. 4, 1523 – 1541
http://dx.doi.org/10.11594/ijmaber.07.04.06

Research Article

Enhancing Pre-service Teachers' Procedural Fluency and Adaptive Reasoning in Trigonometry

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Article history:
Submission 13 January 2026
Revised 31 March 2026
Accepted 23 April 2026

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ABSTRACT

This study examined the effects of Productive Struggle–Driven Creative Problem-Solving (PSD-CPS) instruction on the procedural fluency and adaptive reasoning of pre-service mathematics teachers in a trigonometry course at a public university in Malolos City, Bulacan. Using an embedded mixed-methods design, quantitative data from a quasi-experimental approach and qualitative data from reflective logs, observations, and focus group discussions were analyzed. Results showed significant posttest gains in both control and experimental groups. While procedural fluency gains were moderate, PSD-CPS instruction greatly enhanced adaptive reasoning. Participants positively perceived PSD-CPS as effective in improving their trigonometric understanding.

Keywords: *Content knowledge, Creative problem-solving, Mathematical proficiency, Pre-service mathematics teachers, Productive struggle*

Introduction

The role of the pre-service mathematics teachers in the Philippines's reformation of the mathematics education system is indispensable, especially in addressing challenges in numeracy. Due to the concept of trifocalization, the basic education sector, the Department of Education (DepEd), the higher education sector, the Commission on Higher Education (CHEd), grounded with the Teacher Education Council (TEC), and the technical-vocational education sector, the TESDA, work together to provide stronger quality education to the Filipino learners (Lubid, 2024). However, despite the existing collaborations of the CHEd and DepEd on upscaling teacher education and basic education programs, recent international large-scale assessments (ILSA) results, such as the Programme for International Student Assessments (PISA), revealed that between 2018 and 2022, mean mathematics performance within Organization for Economic Co-operation and Development (OECD) countries decreased by 15 points. Specifically, in the Philippines, although a 2.2%-point increase was observed in the mean scores, the country remains in the bottom 10 of the 81 countries that took the assessment (OECD, 2023).

How to cite:
Sison, L. R. C. (2026). Enhancing Pre-service Teachers' Procedural Fluency and Adaptive Reasoning in Trigonometry. *International Journal of Multidisciplinary: Applied Business and Education Research*. *7*(4), 1523 – 1541. doi: 10.11594/ijmaber.07.04.06

The MATATAG Curriculum is a beacon for addressing the learning gaps and challenges identified through these multiple ILSAs within the past five years. For instance, in the mathematics curriculum, the primary goal is to develop a mathematically proficient and critical problem solver to improve future performance in these assessments. Revising the Mathematics curriculum in basic education for the School Year 2024-2025 calls for learners to demonstrate the five interconnected strands of mathematical proficiency – conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition through relevant teaching-learning engagements and activities (Department of Education, 2024). To seamlessly adapt to the curriculum shift, Teacher Education Institutions (TEIs) must ensure that the pre-service mathematics teachers also possess the skills necessary for carrying out the primary goal of the curriculum. Moreover, pre-service mathematics teachers must be equipped with competencies in order to align with the ideals and aspirations of the curriculum.

However, Altarawneh and Marei (2021) discovered that the pre-service mathematics teachers' level of mathematical proficiency is weak, especially in the strands of adaptive reasoning and procedural fluency. This finding was also attested by the exploration of Usman (2020) where pre-service mathematics teachers have weak procedural fluency in solving trigonometric problems. In addition, Baah-Duodu (2022) identified that pre-service mathematics teachers are nearing proficiency in their adaptive reasoning. Both results attest to the difficulties of the pre-service teachers in these two strands of mathematical reasoning, which calls for further strengthening and improvement in the teacher education sector.

One of the common challenges encountered by pre-service mathematics teachers is their difficulty in mastering and teaching trigonometry, as this field is commonly delivered in Junior High School and Senior High School Mathematics subjects. Nabie, et al. (2018) noted that pre-service teachers perceived trigonometry as complex, static, and too theoretical to learn. They also possess limited content knowledge of the fundamental trigonometric concepts, similar to Usman's (2020) and Fi's (2003) findings. In effect, more than half of the pre-service teachers had difficulties constructing mental structures on fundamental concepts, leading to a meaningful understanding of trigonometric tasks. These studies generally revealed that pre-service mathematics teachers demonstrated proficiency below what is expected of them, especially in the utilization of prerequisite skills and the development of appropriate mathematical solutions.

Focusing on the theoretical underpinning of these mathematics education challenges, Findell, et al. (2001) describes adaptive reasoning as the learner's capability for logical and deeper thinking, appropriate reflection, explanation, and justification of a mathematical solution or process. Moreover, adaptive reasoning encompasses the capacity to enable logical and conceptual thinking and to justify solutions. Likewise, procedural fluency focuses on flexibly, accurately, efficiently, and appropriately carrying out mathematical procedures (Zebua & Waruwu, 2022). In addition, procedural fluency includes the seamless execution of mathematical processes in contextual and practical situations. Hence, to become mathematically proficient, a learner must not rely solely on strong conceptual knowledge but on strategically applying mathematical skills and processes toward solving mathematical problems critically. The pre-service mathematics teachers' procedural fluency and adaptive reasoning are vital in shaping the modern direction of the basic education curriculum, as they are being trained and cultivated to educate future generations of Filipino learners. It is a necessity that they must possess the necessary knowledge, skills, and dispositions, leading to mathematical proficiency.

To address the instructional gaps for pre-service mathematics teachers in adaptive reasoning, Karatas (2022) discovered that supporting productive struggle teaching practice enhances students' procedural fluency. Productive struggle occurs when learners exert labor on a mathematical problem, but still, they want to abandon an inappropriate strategy for another strategy and persevere in seeking solutions until a more sensible solution arrives. For the productive struggle to transpire inside the mathematics classroom, Warshauer (2015) emphasized that teachers must ask significant questions on identifying struggle sources, encourage student reflection on appropriate processes, help students manage their struggles without too much scaffolding, and accept that encountering struggles are vital facets of mathematical learning. In addition, Failing (2024) posited that when pre-service teachers were provided with opportunities in productive struggle-driven instruction, these strengthened their insights towards the pedagogy. Their exposure led to improved understanding and mathematical skills and asking meaningful questions. Sams (2021) also agreed that the utilization of productive struggle instruction within pre-service mathematics teachers decreased their mathematics anxiety and allowed them to perform better in standardized assessments.

Moreover, Stanford (2022) recommended the inclusion of the Creative Problem-Solving (CPS) Model in the instruction, where the utilization of divergent and convergent thinking processes was highlighted in seeking solutions to a problem. Four major stages are reflected in the Creative Problem-Solving Model, which includes (1) clarification of the problem, (2) brainstorming, (3) evaluation/ selection, and (4) implementation of the solution (Pepkin, 2004). However, Andrade and Pasia (2020) discovered that pre-service mathematics teachers demonstrate moderate level of mathematical creativity, of which, the employment of the CPS Model in the instruction is highly encouraged. On the other hand, when aided with graphing software, pre-service teachers can apply series of processes such as specializing, conjecturing, generalizing, and verifying, whenever prompted with open-ended problems (Daher & Anabousy, 2020). Another positive benefit attributed to the CPS model of instruction is the improved teamwork skills and academic achievement of pre-service teachers, embedded with cooperative learning framework, as justified by Jirasak (2017). For instance, this can be employed in a Trigonometry class by presenting a word problem to the class, allowing brainstorming to be fostered in a group of which the group will select and decide on the best solution to the problem. Afterwards, the groups will share their solutions until the teacher provides them with scaffolds and guide questions that will lead them to the verification of the correct and appropriate solution for the problem.

Efforts on pre-service teacher preparation programs were directly put into practice by TEIs, locally and internationally. For instance, Alrwaished (2024) explored the experiences of pre-service Science and Technology, Engineering, and Mathematics (STEM) teachers in a lesson planning training session, where they lacked interdisciplinary conceptual understanding and sufficient guidance from role models or mentors, leading to exposure of pedagogical approaches in mathematics. Moreover, Conde (2014) developed an intervention program called the Didactics of Mathematics Course (DMC) to address the low level of mathematics pedagogical content knowledge (MPCK) of pre-service mathematics teachers, where after their exposure to the program, they demonstrated significant improvement and provided substantial professional opportunities in improving instructional practices. Ebio (2022) also identified problems among pre-service elementary mathematics teachers during a teacher training program. Their lack of mastery of basic mathematical concepts was shown in their limited skills in translating mathematical statements into equations, application of inverses, poor reading comprehension, and inappropriate use of unit conversion.

The importance of teacher preparation programs on the development of mathematical proficiency was emphasized by Beach et al. (2020), emphasizing that pre-service mathematics teachers must be given sufficient opportunities to comprehend problem-solving processes that lead to the understanding and strengthening of mathematical concepts. Hence, it is best that during mathematics teacher preparation programs, pre-service teachers must be given sufficient exposure to improving adaptive reasoning and procedural fluency through problem-solving-led activities and pedagogies.

In addition, since there is a knowledge gap in the fusion of creative problem-solving and productive struggle teaching practices as innovative instruction, the researcher will embed this approach in teaching trigonometry for pre-service mathematics teachers. In addition, this literature led to the research gaps, which prompted the researcher to explore these two strands of mathematical proficiency, namely adaptive reasoning and procedural fluency, among pre-service mathematics teachers in a Trigonometry course. There is an underdevelopment of adaptive reasoning and procedural fluency in the discourse and context of Trigonometry for pre-service mathematics teachers. Nonetheless, there is a need for a general method that prepares pre-service teachers with both theoretical knowledge and practical experiences to foster student-centered learning.

In line with the research gaps and literature analyzed, this research aimed to foster pre-service mathematics teachers' procedural fluency and adaptive reasoning in trigonometry through a Productive Struggle Driven – Creative Problem-Solving (PSD-CPS) instruction. Moreover, the researcher targeted to describe the level of procedural fluency and adaptive reasoning of pre-service mathematics teachers before and after the implementation of the PSD-CPS, and traced whether there were significant differences among their performance levels. Similarly, the learners' perception and experiences in their procedural fluency and adaptive reasoning towards the technique were also identified.

Statement of the Problem

The general problem of this study was: "How may the procedural fluency and adaptive reasoning of the pre-service mathematics teachers in Trigonometry be fostered in a Productive Struggle Driven – Creative Problem-Solving (PSD-CPS) instruction?"

Specifically, this study sought answers to the following questions:

1. How may the pretest and posttest results on procedural fluency of the respondents who were taught and not taught through Productive Struggle Driven – Creative Problem-Solving (PSD-CPS) be described?
2. How may the pretest and posttest results on adaptive reasoning of the respondents who were taught and not taught through Productive Struggle Driven – Creative Problem-Solving (PSD-CPS) be described?
3. Are there significant differences in the pretest and posttest results of the following aspects of mathematical proficiency of the respondents who were taught and not taught through Productive Struggle Driven – Creative Problem-Solving (PSD-CPS)
   3.1. Procedural Fluency
   3.2. Adaptive Reasoning
4. How did a Productive Struggle Driven – Creative Problem-Solving (PSD-CPS) help enhance the respondents' adaptive reasoning and procedural fluency?

Materials and Methods

Research Design

This research employed the embedded mixed methods approach. According to Almeida, Gaerlan and Manly (2016), embedded mixed method research employs both quantitative and qualitative data simultaneously to provide answers to clusters of questions. Likewise, since a single data was not sufficient to answer the general and specific research questions, it was fit to utilize varied data sources collected periodically during the conduct of the experimentation (Creswell & Clark, 2017). During the conduct of the Productive Struggle Driven – Creative Problem-Solving (PSD-CPS) instruction, multiple sources of data were obtained from pre-service teachers' session worksheets, reflections, focus group discussions, and summative assessments evaluated through a rubric for procedural fluency and adaptive reasoning.

Population and Sample

The respondents in the study were all first-year pre-service mathematics teachers who were enrolled during the Academic Year 2024-2025 for the First Semester in the course Trigonometry in a target university in the City of Malolos, Bulacan.

Considering the population of the entire College of Education, this researcher opted for the entire 39 first-year pre-service mathematics teachers to participate in the study since the targeted content area of Trigonometry was intended under the program Bachelor of Secondary Education major in Mathematics curriculum for first-year students, employed through a total enumeration sampling. Nonetheless, they were exposed to the experimental and control treatments in this paper.

Research Instruments

Quantitative data were collected using the Trigonometry Achievement Test (TAT), a 10-item assessment adopted from Hokor and Arhin (2021) that evaluates right triangle trigonometry, bearings, and angles of elevation/depression (α = .708). Students' TAT responses were scored using two adopted tools: a holistic Procedural Fluency Rubric (Aguilar & Telese, 2018; G-coefficient = .92) and an analytic Adaptive Reasoning Rubric (Ansari, et al., 2020) which measures five indicators, including conjecture formulation, pattern recognition, and argument validity.

Qualitative data regarding students' instructional experiences were gathered through a 6-item weekly Reflective Log and a 9-question Semi-structured Focus Group Discussion Guide. Both qualitative instruments underwent rigorous content and technical validation by a panel comprising a mathematics education expert, a college department head, and an English language specialist.

To facilitate the intervention, the researcher developed weekly PSD-CPS Lesson Exemplars, which aligned topics with the PSD-CPS framework and were validated by mathematics education experts. These were supplemented by weekly PSD-CPS Trigonometry Worksheets designed to continuously track and encapsulate students' improvements in problem-solving, explanation, and justification.

Data Collection Procedure

Specifically, the study underwent two phases, including (1) preparation of relevant instructional materials, student orientation and discussion of their ethical safeguards, and pre-testing of the pre-service teachers on Trigonometry and (2) implementation of the traditional and PSD-CPS instruction and post-testing of the pre-service teachers on Trigonometry.

The study was conducted in two primary phases. Phase 1 focused on preparation and pre-testing. Following student orientation and a briefing on ethical safeguards, the researcher prepared and validated the necessary PSD-CPS instructional materials and data-gathering instruments. Participants were then given 1.5 hours to complete the Trigonometry Achievement Test as a pre-test. This assessment measured baseline procedural fluency and adaptive reasoning to ensure initial equivalence before participants were assigned to either the experimental group (PSD-CPS instruction) or the control group (traditional instruction).

Phase 2 comprised the instructional implementation and post-testing. Both groups received equal instructional time, consisting of 1.5-hour sessions. The control group was taught using traditional, CHED-aligned methods explicitly stated in the course syllabus, such as interactive discussions, board work, and speed tests. Conversely, the experimental group was taught using the PSD-CPS instructional framework. Following the intervention, post-tests were administered to both groups to evaluate and compare the effectiveness of the respective teaching methods.

Over the five-week intervention, students completed weekly Reflective Logs and PSD-CPS Worksheets, allowing the teacher to continuously monitor their procedural fluency and adaptive reasoning. To ensure instructional fidelity, the mathematics program head conducted classroom observations for both the experimental and control groups.

Following the final session, both groups were given 1.5 hours to complete the Trigonometry Achievement Test as a post-test. Afterward, a semi-structured focus group discussion involving five students from each group was conducted to explore their learning experiences. The gathered data were then subjected to quantitative and qualitative analyses.

On the other hand, to implement the PSD-CPS instruction, the researcher considered fusing the productive struggle and the creative problem-solving approach in instruction. The researcher will prepare lesson exemplars anchored on the parts of the instructional model. The developmental activities of the lesson exemplars will be focusing on the four stages of creative problem-solving approach, of which the transitions from one stage to another is supported by productive struggle driven practices. Figure 1 shows the researcher-made diagram synthesizing the instructional model used in line with the PSD-CPS instruction, which served as the instructional flow on how was the PSD-CPS carried out inside the trigonometry class.

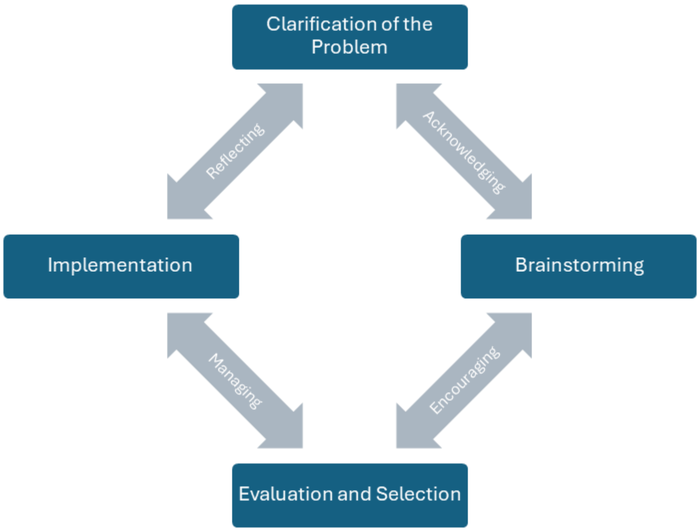

Figure 1. Productive Struggle Driven – Creative Problem-Solving Instruction

The Productive Struggle-Driven Creative Problem Solving (PSD-CPS) instruction is executed through a structured, eight-phase process. It begins with the Clarification of the Problem, where students are divided into collaborative groups and tasked with developing both numerical and verbal mathematical solutions to a given word problem. To prepare them for the task, the facilitator enters the Acknowledging phase, normalizing difficulty by explicitly stating that struggling is a natural and vital part of learning mathematics. With this mindset established, students move into Brainstorming, engaging in open mathematical discourse to generate multiple potential solutions. During this time, they are encouraged to suspend judgment to maximize creative ideas while the facilitator actively monitors their discussions.

As students navigate the problem, the facilitator initiates the Encouraging phase by guiding their thinking processes, helping them identify the root of their struggles, and prompting them to explore alternative approaches. Groups then transition to Evaluation and Selection, where they critically assess their brainstormed ideas, select the most appropriate mathematical solution, and complete their worksheets. Crucially, this is followed by the Managing phase, where the facilitator deliberately steps back. By providing students ample time to work through their difficulties independently without premature intervention, the facilitator promotes intellectual accountability and resilience.

The instructional sequence culminates in the Implementation and Reflecting phases. During implementation, randomly selected groups present their mathematical processes and logical reasoning to the class. The facilitator provides constructive feedback and reveals the standard solution for comparison, cementing the targeted conceptual learning outcome. Finally, the session concludes with students completing their weekly reflection logs. In this reflective phase, the facilitator emphasizes the importance of evaluating the learning effort and the problem-solving journey (the "how") rather than simply focusing on the final answer (the "what").

Ethical Consideration

The consent of the pre-service mathematics teachers was properly sought before the assessment was conducted and the implementation of the proposed instruction. To ensure that sensitive personal information was secured, consents were acquired from the key respondents of this study. The pre-service teachers' approval to participate in the study was obtained through written informed consent, including this study's salient features.

The anonymity of the respondents was also considered. The researcher considered pre-service teachers who opted not to participate in the study. In this manner, the honest willingness of the pre-service teachers to be included in the study was given the utmost respect. Moreover, if the pre-service teacher wishes to withdraw from the study at any point, the researcher respectfully considered this. Nonetheless, the respondents must be assured that the data collected from this study will solely be utilized for research and academic purposes only.

Data Analysis and Statistical Treatment

Descriptive statistics, such as frequency, percentage, mean, and standard deviation, were used to determine the level of procedural fluency of the students through the Trigonometry Achievement Test (TAT) assessed through the Procedural Fluency Rubric and through the weekly results of their worksheets reported through the Productive Struggle Driven – Creative Problem-Solving Worksheet. This corresponded to the second specific question of the study.

Generally, in capturing the students' procedural fluency for the entire period covering the topics on trigonometry, including trigonometric ratios, special trigonometric angles, right triangle trigonometry, and their applications, the TAT Score on Procedural Fluency shown in Table 1, were utilized. Since the cut-off score for passing the course requirements is 75%, this grading scheme will be adopted in the interpretation of their scores, falling in the satisfactory level. This tool provided a deeper holistic analysis of the procedural fluency of the students.

Table 1. Trigonometry Achievement Test (TAT) Score on Procedural Fluency and Verbal Interpretation

Report Mean Score per Item	Overall Mean Score	Level of Procedural Fluency
0 – 1.49	0.00 – 14.49	Unsatisfactory
1.50 – 2.99	14.50 – 29.49	Needs Improvement
3.00 – 3.33	29.50 – 33.49	Satisfactory
3.34 – 3.67	33.50 – 37.49	Highly Satisfactory
3.68 – 4.00	37.50 – 40.00	Outstanding

Like procedural fluency, descriptive statistics such as frequency, percentage, mean and standard deviation were the statistical instruments used to identify the level of adaptive reasoning of learners before and after subjected to the instruction. This determined the pre-service teachers' capacity to think logically about the relationships among trigonometric concepts, situations, and formulas. These descriptive statistical measures indicated the characteristics of the adaptive reasoning, which led to a better understanding of the problem. The scale, shown in Table 2, will also be applied to identify its corresponding level. Following the cut-off consideration for procedural fluency, 75% of the total points will fall to the satisfactory level. This corresponded to the third specific question of the study.

Table 2. Trigonometry Achievement Test (TAT) Score on Adaptive Reasoning and Verbal Interpretation

Report Mean Score per Item	Overall Mean Score	Level of Adaptive Reasoning
0.00 – 6.49	0.00 – 64.99	Unsatisfactory
6.50 – 14.49	65.00 – 144.99	Needs Improvement
14.50 – 16.49	145.00 – 164.99	Satisfactory
16.50 – 18.49	165.00 – 184.99	Highly Satisfactory
18.50 – 20.00	185.00 – 200.00	Outstanding

Quantitative data were analyzed using inferential statistics following normality checks via the Shapiro-Wilk test. To establish baseline equivalence and compare between-group differences (experimental vs. control), Independent samples t-tests were utilized, with the Mann-Whitney U test applied for non-normally distributed data. To measure within-group improvements from pre-test to post-test, dependent t-tests or Wilcoxon Signed-Rank tests were employed (Alferez & Duro, 2018). Furthermore, effect sizes were calculated and interpreted based on Pallant's (2020) criteria (r = .1 for small, .3 for medium, .5 for large).

For the qualitative data, narrative and categorical-content analysis (Komolthiti, 2016) were utilized to extract core themes from the students' written reflective logs, the teacher-observer's logs, and focus group discussions. After the coding process was validated by an independent education expert, the qualitative themes were triangulated against the quantitative findings. This integrated approach allowed for a comprehensive understanding of the participants' experiences and the overall effectiveness of the PSD-CPS instruction (Saldaña, 2021).

Result and Discussion

Level of Procedural Fluency of Pre-service Teachers in Trigonometry

Table 3. Level of Procedural Fluency of the Pre-service Teachers

Item Number	Control Group						Experimental Group					
	Pre-test			Post-test			Pre-test			Post-test		
	Mean	SD	VI	Mean	SD	VI	Mean	SD	VI	Mean	SD	VI
1	2.94	0.96	NI	3.74	0.79	O	2.84	1.17	NI	4.00	0.00	O
2	0.53	0.76	U	3.74	0.79	O	0.63	0.76	U	3.84	0.69	O
3	1.21	1.21	U	3.89	0.45	O	1.68	1.49	U	3.94	0.23	O
4	0.95	0.96	U	3.68	0.81	O	0.68	1.29	U	3.58	1.02	HS
5	0.32	0.47	U	3.63	0.88	HS	0.63	1.26	U	3.84	0.69	O
6	0.00	0.00	U	1.47	0.95	U	0.00	0.00	U	1.32	1.00	U
7	0.05	0.23	U	2.37	1.62	NI	0.11	0.31	U	3.11	1.45	S
8	0.21	0.41	U	0.89	0.45	U	0.00	0.00	U	1.37	1.01	U
9	0.37	0.67	U	1.58	0.76	NI	0.53	0.84	U	1.89	0.88	NI
10	0.00	0.00	U	3.26	1.50	S	0.00	0.00	U	2.74	1.76	NI
Overall Level	**6.58**	**3.04**	**U**	**28.26**	**4.89**	**NI**	**7.11**	**4.46**	**NI**	**29.63**	**3.89**	**S**

Note: Maximum points per item is 4. Overall items of procedural fluency are 40.
Legend: U – unsatisfactory, NI – needs improvement, S – satisfactory, HS – highly satisfactory, O – outstanding

Table 3 presents the comparative levels of procedural fluency for both the control and experimental groups before and after their respective instructions. During the pre-test, both groups demonstrated an unsatisfactory to needs-improvement initial grasp of procedural fluency (Control overall $\bar{x}$ = 6.58; Experimental overall $\bar{x}$ = 7.11). Item-level analysis revealed that students across both groups initially struggled with applied concepts, particularly problems involving angles of elevation and depression (Items 6 and 10), as well as bearing problems (Item 8), which frequently yielded pre-test mean scores of zero.

Following the five-week interventions, both groups showed significant score increases. However, the experimental group exposed to Productive Struggle Driven–Creative Problem-Solving (PSD-CPS) instruction advanced to a Satisfactory overall level ($\bar{x}$ = 29.63, SD = 3.89), whereas the control group exposed to traditional instruction remained at the Needs Improvement level ($\bar{x}$ = 28.26, SD = 4.89). The greatest gains for the experimental group were seen in evaluating basic trigonometric expressions and special angles (Items 1, 2, 3, and 5), where students achieved Outstanding levels of fluency. Despite these gains, applied bearing problems (Item 8) remained a relative challenge (Experimental $\bar{x}$ = 1.37; Control $\bar{x}$ = 0.89), indicating that appropriate spatial visualization in complex applications requires ongoing instructional support.

The quantitative improvements in the experimental group are strongly supported by the students' qualitative feedback. Focus group discussions and written reflections revealed that students attributed their success to the PSD-CPS framework, noting that it specifically helped them "break down problems into a step-by-step process," "visualize each problem appropriately," and "identify the accurate procedure to use". This thematic alignment directly reflects the National Council for Teachers of Mathematics (NCTM) standards, which define strong procedural fluency as the accurate use of procedures, flexibility in strategy, and the ability to locate errors in one's own solutions (Altarawneh & Marei, 2021). Furthermore, as Aguilar and Telese (2018) noted, a procedurally fluent pre-service teacher demonstrates a command of mathematical processes that allows them to navigate procedures seamlessly.

Comparatively, these results reinforce existing literature on the benefits of non-traditional problem-solving instructions. Fortes and Andrade (2019) emphasize that teachers must encourage students to think critically and flexibly rather than mechanistically when solving problems. By utilizing the PSD-CPS instruction, pre-service teachers in the experimental group were pushed to engage creatively with trigonometric concepts. This led to a deeper conceptual understanding and a more effective application of procedural fluency, corroborating the findings of Annurwanda and Friantini (2022), who similarly concluded that creative problem-solving models significantly enhance students' procedural capabilities

Level of Adaptive Reasoning of Pre-service Teachers in Trigonometry

Table 4. Level of Adaptive Reasoning of the Pre-service Teachers

Item Number	**Control Group**						**Experimental Group**					
	Pre-test			**Post-test**			**Pre-test**			**Post-test**		
	Mean	**SD**	**VI**	**Mean**	**SD**	**VI**	**Mean**	**SD**	**VI**	**Mean**	**SD**	**VI**
1	12.37	3.48	NI	15.68	3.01	S	11.74	3.90	NI	16.37	1.50	S
2	2.53	3.52	U	15.42	2.91	S	3.05	3.47	U	16.00	2.69	S
3	5.37	4.50	U	16.16	2.22	S	7.32	6.20	U	16.37	1.30	S
4	4.58	4.46	U	15.58	3.01	S	2.47	4.30	U	15.37	3.61	S
5	1.68	2.58	U	14.79	3.07	S	2.52	4.35	U	15.42	2.93	S
6	0.00	0.00	U	8.11	3.16	NI	0.00	0.00	U	7.84	3.48	NI
7	0.26	1.15	U	11.37	6.36	NI	0.53	1.58	U	13.84	5.12	NI
8	1.05	2.09	U	6.05	2.99	U	0.00	0.00	U	7.95	3.47	NI
9	1.84	3.42	U	7.32	3.73	NI	2.63	4.21	U	8.95	4.13	NI
10	0.00	0.00	U	13.89	6.63	NI	0.00	0.00	U	11.79	7.38	NI
Overall	**29.68**	**2.93**	**U**	**124.37**	**5.40**	**NI**	**30.26**	**17.09**	**U**	**129.89**	**18.14**	**NI**

Note: Maximum points per item is 20. Overall items of adaptive reasoning are 200.

Legend: U – unsatisfactory, NI – needs improvement, S – satisfactory, HS – highly satisfactory, O – outstanding

Table 4 illustrates the comparative levels of adaptive reasoning between the control group (exposed to traditional interactive discussion and board work) and the experimental group (exposed to PSD-CPS instruction). During the pre-test, both groups demonstrated an overall Unsatisfactory level of adaptive reasoning. Item-level analysis revealed striking similarities in their baseline knowledge: both groups only managed a "Needs Improvement" rating on Item 1, which relies heavily on prior knowledge of basic trigonometric ratios from their Junior and Senior High School curricula. Furthermore, no pre-service teachers in either group even attempted Items 6 and 10 (angles of elevation and depression). This shared initial deficiency highlights a severe lack of familiarity with visualizing worded problems and moving beyond mere computation to actual logical justification.

Following the five-week interventions, both groups showed notable score increases, though both ultimately achieved an overall Needs Improvement level for adaptive reasoning. Despite falling short of an overall satisfactory mark, differences emerged between the groups' conceptual development. Both groups performed best at evaluating standard trigonometric expressions (Item 3), reaching satisfactory levels of justification. However, their specific areas of struggle differed. The control group's reasoning remained firmly Unsatisfactory on applied bearing problems (Item 8) due to erroneous justifications regarding the appropriate rotation of angles on a north-south line. The experimental group, while also challenged by applied problems, managed to elevate their lowest-performing item (Item 6, angle of depression) to a "Needs Improvement" level. Their primary remaining challenge was not a complete lack of understanding, but rather insufficient visual representation and missing logical links connecting midpoint concepts to their final solutions.

These comparative findings closely align with existing pedagogical research regarding instructional methods and higher-order thinking. The limited progress of the control group corroborates Mohammed's (2021) observation that while traditional interactive discussions can improve basic conceptual application, they frequently fall short in developing the deeper adaptive reasoning required for complex problems. As Eslit (2023) argued, traditional methods alone are insufficient for fostering adaptive reasoning and must be combined with dynamic, inquiry-based strategies.

Conversely, the experimental group's targeted improvements support the use of structured problem-solving interventions. Gurat (2018) demonstrated that problem-solving frameworks in mathematics significantly enhance preservice teachers' capacity to explicitly justify and connect mathematical concepts. Likewise, Widodo and Nurpatri (2020) reported that creative problem-solving models are highly effective in developing adaptive reasoning, particularly for tasks requiring logical and conceptual integration. While a five-week PSD-CPS intervention was not entirely sufficient to push the students' overall adaptive reasoning to a satisfactory level, it provided a stronger framework for addressing reasoning deficiencies and visualization challenges than traditional instruction.

Significant Differences of Adaptive Reasoning and Procedural Fluency Before and After the Instruction

Table 5. Test of Difference between Pretest Scores of Students in terms of Procedural Fluency of Experimental and Control Group

Group	**N**	**Mean Rank**	**Sum of Ranks**	**U**	**p-value**
Experimental	19	19.32	367.00	177.00	.931
Control	**19**	**19.68**	**374.00**		

Because the data violated normality assumptions, a Mann-Whitney U test was used to compare the pretest procedural fluency scores of the experimental and control groups (n = 19 each). Results indicated no significant difference between the control (mean rank = 19.68) and experimental groups (mean rank = 19.32; U = 177.000, p = .931). This confirms baseline equivalence between the two groups prior to the intervention. Establishing this initial comparability is essential, as it minimizes the influence of extraneous variables (Thomas, 2023) and ensures that any post-treatment differences can be confidently attributed to the instructional methods rather than preexisting disparities (Larbi, 2016).

Table 6. Test of Difference between Pretest and Posttest Scores of Students in terms of Procedural Fluency of Control and Experimental Group

Group	Pretest Median	Post-test Median	*Z*	Asymp. Sig. (2-tailed)	r
Control	7.00	30.00	-3.706*b*	.000	.60
Experimental	**6.00**	**32.00**	**-3.828*b***	**.000**	**.62**

Note: Sample Size (n = 19); *significant at α = 0.05; effect size *r* (Cohen's d) value 0.20 (small effect), 0.50 (medium effect), and 0.80 (large effect); *b*: based on positive ranks

Table 6 reveals that a Wilcoxon Signed Rank Test showed that there was a significant difference (Z = -3.706, p = .000) between scores given for the pre-test of the Trigonometry Achievement Tet (TAT) evaluated through the Procedural Fluency Rubric (Mdn = 7.00) compared to the post-test (Mdn = 30.00), which measures the procedural fluency of the students in the control group. In addition, the same observation is considered with the experimental group where the pre-test scores (Mdn = 6.00) significantly differ (Z = -3.828, p = .000) with the post-test scores (Mdn = 32.00). Therefore, both the traditional and the Productive Struggle Driven – Creative Problem Solving (PSD-CPS) instruction has been proven effective in improving the students' procedural fluency in the respective group in which the instruction is implemented. Both groups demonstrated a medium effect in improving the procedural fluency scores, as revealed by the effect size *r*.

Teachers should encourage their students to be more flexible and think more critically when solving problems (Fortes & Andrade, 2019). Thus, by using the PSD-CPS instruction, students were encouraged to engage more creatively with mathematical concepts, leading to a deeper understanding and more effective application of procedural fluency, similar to the findings of Annurwanda and Friantini (2022).

Table 7. Test of Difference between Posttest Scores of Students in terms of Procedural Fluency of Experimental and Control Group

Group	N	Mean Rank	Sum of Ranks	U	p-value
Experimental	19	20.97	398.50	152.500	.418
Control	**19**	**18.03**	**342.50**		

A Mann-Whitney U test revealed no significant difference in posttest procedural fluency scores between the experimental (mean rank = 20.97) and control groups (mean rank = 18.03; *U* = 152.500, *p* = .418). This indicates that both PSD-CPS and traditional instruction were equally effective in improving procedural fluency. This finding aligns with Mohammed (2024), who noted that both traditional and inquiry-based teaching methods can successfully enhance mathematical skills. However, it contrasts with Tursynkulova et al. (2023), who found that innovative, problem-solving-based approaches typically yield significantly greater procedural fluency than traditional methods.

Table 8. Test of Difference between Pretest Scores of Students in terms of Adaptive Reasoning of Experimental and Control Group

Group	n	Mean	SD	t	Df	p-value
Experimental	19	30.26	17.09	.118	36	.907
Control	**19**	**29.68**	**12.78**			

An independent t-test compared the pretest adaptive reasoning scores of the experimental ($\bar{x}$ = 30.26, SD = 17.09) and control ($\bar{x}$ = 29.68, SD = 12.78) groups, each consisting of 19 pre-service teachers. Results revealed no significant difference between the two groups, $t(36) = 0.118$, $p = .907$. Establishing this baseline equivalence ensures that any post-intervention outcomes can be reliably attributed directly to the treatment rather than pre-existing disparities, fulfilling a crucial requirement for meaningful experimental comparisons (DeCarlo, Cummings, & Agnelli, 2020; Thomas, 2023).

Table 9. Test of Difference between Pretest and Posttest Scores of Students in terms of Adaptive Reasoning of Control Group

Group	Pretest Median	Post-test Median	*Z*	Asymp. Sig. (2-tailed)	*r*
Control	**29.00**	**136.00**	**3.823*b**	**.000**	**.62**

Note: Sample Size (n = 19); *significant at α = 0.05; effect size *r* (Cohen's d) value 0.20 (small effect), 0.50 (medium effect), and 0.80 (large effect); *b*: based on positive ranks

Based on the Table 9, a Wilcoxon Signed Rank Test showed that there was a significant difference (Z = 3.823, p < .05) between scores given for the pre-test of the Trigonometry Achievement Test (TAT) assessed through the Adaptive Reasoning Rubric (Mdn = 29.00) compared to the post-test (Mdn = 136.00), which measures the adaptive reasoning of the pre-service mathematics teachers in trigonometry. Therefore, the use of traditional instruction approaches for the control group, has shown to be effective in improving the adaptive reasoning of the pre-service teachers in right triangle trigonometry. Furthermore, a medium effect was computed ($r = .62$) where it is justified that the time-tested and research-based traditional approaches such as interactive discussions, board works, answering exercises, and viewing relevant video lessons improve the adaptive reasoning in the field. Aligned to these traditional instructional approaches, Pulls and Burns (2022) support this finding, emphasizing that concrete-representational-abstract, feedback, peer learning strategies, self-monitoring, and self-regulated learning have a statistically significant positive effect on fostering adaptive reasoning in mathematics classrooms, further validating the effectiveness of traditional approaches in improving pre-service teachers' adaptive reasoning skills in right triangle trigonometry. This connection is further supported by Standford (2022), whose examination of the Problem-Based Learning (PBL) model highlighted how engaging students in real-life problem-solving within collaborative groups also promotes the development of adaptive reasoning skills, illustrating the complementary value of traditional and innovative instructional approaches.

Table 10. Test of Difference between Pretest and Posttest Scores of Students in terms of Adaptive Reasoning of Experimental Group

Test	n	Mean	SD	t	df	p-value	*r*
Pretest	19	30.26	17.09	20.041	18	.000	4.70
Posttest	**19**	**129.89**	**18.13**				

A dependent-sample t-test was facilitated to determine whether adaptive reasoning improved using the Productive Struggle Driven – Creative Problem-Solving (PSD-CPS) instruction on right triangle trigonometry. The results showed that the average adaptive reasoning scores before the proposed instruction ($\bar{x} = 30.26$, SD = 2.06) were significantly lower than the average math fact fluency scores obtained after the exposure ($\bar{x} = 129.89$, SD = 18.13) to the said instruction ($t(18) = 20.041$, $p < .000$). This implies that the PSD-CPS instruction was effective to be utilized in improving the adaptive reasoning of the pre-service mathematics teachers in trigonometry. Moreover, the effect size *r* was determined to identify the extent on how the said intervention affected the math fact fluency of the students. A large effect (r = 4.70) was identified, which signifies a highly impactful conduct of instruction to the pre-service teachers' adaptive reasoning in right triangle trigonometry. Similar findings were reported by Muin et al. (2022), showing that the Creative Problem-Solving (CPS) model effectively fosters mathematical adaptive reasoning skills of the students by encouraging innovation and exploration in problem-solving. Their study reflects the comprehensive enhancement of adaptive reasoning using CPS model, as evidenced also by the findings of Ansari et al. (2020), further reinforcing the effectiveness of the PSD-CPS instruction in this study.

Table 11. Test of Difference between Posttest Scores of Students in terms of Adaptive Reasoning of Control Group and Experimental Group

Group	N	Mean Rank	Sum of Ranks	U	p-value
Experimental	19	19.95	379.00	172.000	.817
Control	**19**	**19.05**	**362.00**		

A Mann-Whitney U test revealed no significant difference in posttest adaptive reasoning scores between the experimental (mean rank = 19.95) and control groups (mean rank = 19.05; U = 172.000, p = .817). This indicates that both PSD-CPS and traditional instruction were equally effective in improving adaptive reasoning. These results contrast with previous research (Komarudin et al., 2023), which found that Creative Problem-Solving (CPS) learning models are significantly more effective than conventional methods in fostering students' adaptive reasoning.

Effects of the PSD-CPS Instruction on the *Adaptive Reasoning and Procedural Fluency of Pre-service Mathematics Teachers in Trigonometry*

To properly encapsulate the qualitative responses from the conduct of the intervention, the researcher inspected the semi-structured focus group discussion for selected participants in the experimental group, including the written reflection of student-respondents. These analyses were conducted to determine how did the PSD-CPS instruction support the pre-service mathematics teachers improve their adaptive reasoning and procedural fluency along the implementation.

Table 12. Semi-structured Focus Group Discussion and Written Reflection Responses focused on Procedural Fluency

Themes	Sub-themes	Codes	Sample Responses
Use of PSD-CPS instruction contributes to the enhancement of procedural fluency.	Effects of PSD-CPS instruction on pre-service teachers' procedural fluency	APPROPRIATE VISUALIZATION[1]	Student 9: "…was able to visualize each problem appropriately…"
		ACCURATE SOLUTION[3]	Student 4: "…formulate [an] accurate solution from the given problem…"
		RIGHT FORMULAS[1]	Student 7: "… use of trigonometric ratios and Pythagorean Theorem…"
		ACCURATE PROCEDURE[3]	Student 10: "…identify the accurate procedure to use…'
		BREAKING INTO PROCEDURES[2]	Student 3: "…breaking down problems into step-by-step process…"

Note: 1 – Descriptive Coding; 2 – Process Coding; 3 – In Vivo Coding

As detailed in Table 12, the examination of students' semi-structured focus group discussion responses and written reflections, primarily focused on procedural fluency, were interpreted under suitable themes. This analysis utilized descriptive, process, and in vivo coding methods, culminating in a deductive coding approach.

The students in the experimental group exhibited positive results, attributed to implementing the Productive Struggle Fluency – Creative Problem-Solving (PSD-CPS) instruction. Observable advancements in their adaptive reasoning were evident through multiple indicators: (1) had visualized problems appropriately, (2) displayed accurate solutions during collaborative sessions, (3) utilized the right formulas for a specific problem, (4) delineated the appropriate procedure to apply in a particular situation, and (5) specified the procedures relevant in coming up with a solution to the problem. These analyzed themes led to the notion that the PSD-CPD instruction brought significant changes to the procedural fluency of the pre-service mathematics teachers in trigonometry.

The National Council for Teachers in Mathematics (NCTM) identified manifestations of a strong procedural fluency among students which include an accurate use of procedures, demonstration of flexibility in multiple strategies, and use of multiple ways to go through a solution and locating errors in own and other's solution (Altarawneh & Marei, 2021). Moreover, a procedurally fluent pre-service teacher demonstrates command of the processes involved in a particular concept such as trigonometry which makes them navigate procedures from one process to another seamlessly (Aguilar & Telese, 2018). These descriptions match the perceived effects of the pre-service teachers in the experimental group in terms of their procedural reasoning, supporting the claim of the current research.

Table 13. Semi-structured Focus Group Discussion and Written Reflection Responses focused on Adaptive Reasoning

Themes	Sub-themes	Codes	Sample Responses
Use of PSD-CPS instruction contributes to the enhancement of adaptive reasoning.	Effects of PSD-CPS instruction on pre-service teachers' adaptive reasoning	IMPROVED JUSTIFICATION[1]	Student 2: "… has improved in explaining one's group's solution…
		BETTER ANALYSIS[1]	Student 10: "…heightens analyzation and critical thinking skills…"
		SYSTEMATIC[3]	Student 4: "…developed a systematic approach in solving trigonometric problems."
		FINDING PATTERNS[2]	Student 8: "…can recognize and break down patterns."
		DRAWING CONCLUSIONS[2]	Student 1: "…connect with practical problems, … enabling to see how concept[s] fit together."

Note: 1 – Descriptive Coding; 2 – Process Coding; 3 – In Vivo Coding

Table 13 displays the students' semi-structured focus group discussion responses and written reflections, geared towards discovering the effects of the PSD-CPS instruction on their adaptive reasoning, were interpreted under suitable themes. Through a deductive coding approach applied with descriptive, process, and in vivo coding strategies, the general theme of the utilization of the PSD-CPS contributing to the enhancement of adaptive reasoning emerged.

The pre-service teachers in the experimental group demonstrated positive indicators which can be linked to their fostering of their adaptive reasoning in trigonometry. These instances include (1) improvement in justification of one's solution, (2) better analysis of finer details in a word problem, (3) development of a systematic approach when given a trigonometric problem, (4) seeking observable and hidden patterns within problems, and (5) drawing conclusions out of the consolidated mathematical solutions.

A student demonstrating high levels of adaptive reasoning was able to find and rectify errors which consider mistakes as part of their learning, identify appropriate strategies of which their use is justified, provide explanations to lines of solution, transfer learning from one situation to another, and able to ask questions to clarify a particular construct (Altarawneh & Marei, 2021). In addition, the essence of mathematical communication is rooted in adaptive reasoning, where the pre-service teachers were provided opportunities to solve trigonometric problems and justify how they have come up with a particular solution (Bansu, et al., 2020). Nonetheless, these literatures on adaptive reasoning supports the claim of the research on demonstrating the perceived positive effects of PSD-CPS instruction on the adaptive reasoning of the pre-service teachers in trigonometry.

Table 14. Joint Display on the Mixed Data on the Changes in Pre-service Teachers' Procedural Fluency and Adaptive Reasoning

Quantitative Data	**Qualitative Data**	**Meta-Inference**
Overall mean for Procedural Fluency: Pre – 7.11; Post – 29.63 Wilcoxon Signed Rank Test: Z = -3.828, p = .000, r = .62	Student 9: "…was able to visualize each problem appropriately…" Student 4: "…formulate [an] accurate solution from the given problem…" Student 7: "… use of trigonometric ratios and Pythagorean Theorem…" Student 10: "…identify the accurate procedure to use…' Student 3: "…breaking down problems into step-by-step process…"	**Concordance** There is a significant change in the procedural fluency of the pre-service teachers in the experimental group attested by the medium effect size. The qualitative data also reveals noticeable improvements in procedural fluency. The qualitative accounts indicate that the pre-service teachers show an improved perception of their adaptive reasoning after taught through the PSD-CPS instruction.
Overall mean for Adaptive Reasoning: Pre – 29.68, Post – 124.37 Dependent t-test: Z = 20.041, p = .000, r = 4.70	Student 2: "… has improved in explaining one's group's solution…" Student 10: "…heightens analyzation and critical thinking skills…" Student 4: "…developed a systematic approach in solving trigonometric problems." Student 8: "…can recognize and break down patterns." Student 1: "…connect with practical problems, … enabling to see how concept[s] fit together."	**Concordance** There is a significant change in the adaptive reasoning of the pre-service teachers in the experimental group, attested by the large effect size. The qualitative data matches the quantitative data as they reveal observable progress in adaptive reasoning. The qualitative accounts indicate substantial changes in how the pre-service teachers solve trigonometric problems after being exposed to the PSD-CPS instruction.

Conclusion

Based on the study, implementing Productive Struggle Driven – Creative Problem Solving (PSD-CPS) instruction in trigonometry yielded the following results for pre-service teachers:

1. PSD-CPS produced a statistically significant, moderate improvement. It elevated students from "needs improvement" to "satisfactory," outperforming the traditional intervention group which showed no improvement.
2. PSD-CPS yielded a statistically significant, large improvement, raising students from an "unsatisfactory" baseline to "needs improvement." While both the experimental and control groups ultimately reached the same descriptive level, the PSD-CPS group scored slightly higher.
3. Despite the clear benefits of the PSD-CPS method, further academic support is still necessary, as some students failed to reach the 75% proficiency cutoff in both fluency and reasoning.
4. Both quantitative and qualitative data confirm that PSD-CPS is a highly effective, student-centered approach. By encouraging students to select formulas and justify their methods, the instruction creates more efficient, metacognitively aware problem solvers with stronger mathematical communication skills.

Recommendations

In line with the research's salient findings and key conclusions, the researcher presents the following recommendations to the educational community.

1. Provide specific tutoring or refresher sessions for pre-service teachers who still struggle with procedural fluency after the PSD-CPS instruction. Focus these sessions on the exact skills covered in the study to ensure every student meets the CHED learning outcomes.
2. Schedule regular, short assessments throughout the academic year to monitor students' adaptive reasoning. Use this data to continuously apply research-backed teaching methods that strengthen the specific areas where students need the most help.
3. Use the PSD-CPS method to teach core trigonometry concepts. Instructors do not need to start from scratch; they can either directly adapt the verified lesson plans used in this study or use them as a template to build new plans tailored to their classrooms.
4. Administrators and supervisors should actively write PSD-CPS instruction into their new program proposals and intervention plans. Piloting this approach in other departments will help determine if its benefits scale to the broader student body.
5. Future researchers should test how PSD-CPS instruction affects other areas of math proficiency, specifically conceptual understanding, strategic competence, and productive disposition. Additionally, researchers should replicate this study with entirely different student demographics to see if the results hold up.

Acknowledgement

The author would like to express profound gratitude to the College of Professional Teacher Education at Bulacan State University for the institutional support provided during this study. Sincere thanks are also extended to the pre-service mathematics teachers who enthusiastically participated in the research, and to the experts who validated the instruments and data analysis.

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