

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY: APPLIED BUSINESS AND EDUCATION RESEARCH

2022, Vol. 3, No. 11, 2440 – 2450

<http://dx.doi.org/10.11594/ijmaber.03.11.27>

Research Article

Learnings from Conducting an Online Lesson Study for the Online Teaching of Radian Measure

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Article history:

Submission October 2022

Revised November 2022

Accepted November 2022

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ABSTRACT

With the shift of learning modality from face-to-face to online distance learning, teaching Mathematics has been linked to new challenges and constraints. This paper investigates how radian measure can be taught via distance learning delivery modality. An Online Lesson Study was conducted, and the research lesson was implemented for first-year college students taking Bachelor of Secondary Education major in Mathematics at a state university in the Philippines. The recorded dialogues during the post-lesson discussion were transcribed and analyzed by the researchers. Through these processes, three themes emerged: (1) logistical issues in online teaching, (2) alignment of purpose and approach in online teaching, and (3) theoretical underpinnings for online teaching. Hence, online lesson study provides a venue for teachers to pursue professional development amidst the pandemic.

Keywords: *Black Box Effect, Distance Learning, Multimedia Instructional Principles, Online Lesson Study, Radian Measure*

Introduction

Abrupt and unexpected changes in teaching modality from face-to-face to online distance learning because of the pandemic brought about challenges in teaching and learning

mathematics online. Taley et al. (2021) reported that students had low level of online mathematics learning experience which negatively affected their critical inquiry skills and learning outcomes. Consequently, online teach-

How to cite:

Flores, L. A., Presto, A. C., Combo, J. E. A., Caballero, J. G., Palanas, V. D., Devis, R. A., & Elipane, L. E. (2022). Learnings from Conducting an Online Lesson Study for the Online Teaching of Radian Measure. *International Journal of Multidisciplinary: Applied Business and Education Research*. 3 (11), 2440 – 2450. doi: 10.11594/ijmaber.03.11.27

ing of mathematics requires complete and concrete logistical and pedagogical planning for instruction to be implemented in the best way possible. According to Cassibba et al. (2021), there are two factors to consider when teaching mathematics at the postsecondary level using an online modality. The first is connected to the fact that instructors typically teach mathematics by writing formulas and symbols on the board for students to observe. The second part is concerned with the teaching of mathematics based on the student's specialization. Teaching mathematics to students in a mathematics degree course requires a different method than teaching mathematics in another degree course, as was clear even before the pandemic. For instance, instructors tend to employ more direct instruction while teaching trigonometry to students who are not mathematics majors as opposed to giving them an in-depth understanding of the creation or construction of the formula.

In the Philippines, trigonometry is one of the most important major disciplines taught in tertiary mathematics degree programs (such as the Bachelor of Secondary Education major in Mathematics). This course targets a range of cognitive abilities in students and has many real-world applications. To understand and resolve issues involving angles, triangles, and their relationships, it necessitates that students apply their geometric, graphical, and algebraic thinking. Additionally, it has several applications in a variety of academic subjects, including astronomy, geography, cartography, electricity, geometry, maritime, optics, and physics (Nabie et al., 2018).

However, trigonometry is a subject that many find challenging in the classroom (Tatar et al., 2008). Trigonometry was viewed as difficult, abstract (compared to other mathematical courses), and tedious to learn by students and pre-service teachers (Gur, 2009; Nabie et al., 2018). The limited conceptual understanding of fundamental trigonometric principles among pre-service instructors is reflected in this assessment (Nabie et al., 2018).

Degree and radian are the two measures of angles that are used when learning trigonometry in school. Although they have been intro-

duced to degree measurement since elementary school, students learn about radian measuring for the first time in high school. Since the relationship between radian and degree is constantly used and introduced to the students in teaching trigonometry, students can easily translate degrees to radians and vice versa and are aware that $\frac{\pi}{180} = 1^\circ$ and π radians = 180° . However, despite their basic knowledge of two measures of angles, the concept of radians is not understood visually by students (Orhun, 2001; Kamber & Takaci, 2017). This explanation falls short of helping students understand the radian as a ratio of arc and as a radius of a circle (Challenger, 2009). They discovered (from preliminary studies) that students are confused regarding the value of π , whether it is 3.14 or 180° . They claim that π is equal to 3.14 on a circle but π is equal to 180° at an angle, hence there are two values of π .

This implies that the students are aware that radians are related to angles, but they do not fully comprehend the concept. Students' knowledge of more difficult topics such as trigonometric functions suffers because of their lack of understanding of the concept of radians. Without a basic understanding of the radian, graphing and analyzing trigonometric functions, solving trigonometric equations, and determining the limits of trigonometric expressions will be difficult (Walsh et al., 2017). As a result, a better comprehension of the radian leads to other trigonometric concepts.

Online Lesson Study (OLS)

"The essence of Lesson Study (LS) lies in the amount of intellectual and affective engagement of its participants who engender a spirit of collaboration – working on a shared goal that they themselves generated" (Elipane, 2012, p. 7). This idea emphasizes the significance of collaboration through LS when it comes to developing teachers' capacity to facilitate students' learning and to distinguish how the teaching-learning process would be optimized—in this case, addressing the challenges in delivery modality, which is online. Due to the sudden outbreak of CoViD 19, face-to-face interactions among students and teachers are temporarily prohibited, which led to the utilization of online

platforms in implementing lessons, and therefore, researching on lessons, such as OLS. Nevertheless, on the academic paper of Hird et al. (2014) cited by Calleja and Camilleri (2021), it was reiterated that there is a great potential once LS is associated to lesson sharing where they defined the latter as "...sharing of lessons by teachers via the Internet for others to use" (p 3). This notion draws implications on how to address the challenges that this new normal brought to engagement in OLS, especially on how its phases will be conducted since online platforms are already part of the picture to which not all teachers or even students are accustomed.

As mentioned by Goei et al. (2020), transforming and integrating the usual format of LS into an online platform is a challenge that needs to be explored. They also added to their conceptual paper that "...we saw online teaching and learning rapidly developing, together with heavier technology usage. We assume this change is a call for a different balance of teaching interventions and pedagogy to be deployed in LS." (p 2). This goes to show that even though OLS demands technology-based practices, still, teachers' didactical interventions are highly and equally essential. Mishra and Koehler (2006) supported this, as cited by Huang et al. (2021), wherein they stressed that teachers' knowledge and capability in utilizing digital resources and materials in teaching mathematics and as a support on how the subject itself will also be learned are very relevant.

Didactical Phenomena

A priori analysis is one of the most important things to consider in preparation for an LS. It is usually conducted before taking action on a particular problem that a teacher/facilitator would encounter during the implementation of the lesson (Rumanova et al., 2015). Since this LS was implemented virtually, technology-based strategies were deemed necessary to make the session meaningful and participatory. Part of the *a priori* analysis conducted by the researchers was to determine the possible didactical phenomena that could exist as the discussions go online. Some of these so-called phenomena are identified as Jourdain Effect, Metacognitive shift, and the Black Box Effect. After

considering several possible problems, the researchers unanimously identified the latter as the most likely to occur alongside the learners' possible responses.

There are teachers during discussions who tend to not acknowledge the fact that his/her learners sometimes lack idea on an issue simply because they could provide answers that are just based from ordinary causes or a result of similar activities like analogy and others. This scenario is an illustration of Jourdain Effect, a type of Topaze Effect (Clarke et al., 2006). This didactical phenomenon was one of the initial considerations of the researchers since it has something to do with what the teachers would like to expect from their learners during discussions, that is, for them to be proactive. However, because of the intention of the teacher to make his/her discussion a much participative one, there are times that he/she would subconsciously give or hint the answers already and believe it as a manifestation of conceptual learning.

Another possible phenomenon that the researchers had predetermined during planning and designing phase was the Metacognitive Shift. As part of teaching process, it is expected that there are some activities that may not work the way the teacher would like to because the students cannot remember some basic concepts. Hence, as a way to continue the activity, the teacher would usually inject discussion of previous topics including his/her own formulations and ideas which shift the teaching-learning processes into a different discussion (Brousseau, 2006).

As per the study of Winslow (2003), he mentioned an example where the Black Box Effect is generally determined whenever there is a linear relationship between the learners' input and its corresponding output, which means the process of getting the correct response is not explicitly shown. This underpinned the concept of the Black Box Effect to be well-known for not being 'process oriented,' whereas it does not put so much emphasis on possible interventions (Carlgren, 2012). Subsequently, the facilitator of this current LS used an online assessment tool for which the result is determined to right after the learners' answer. The phenomenon mentioned above is

quite difficult to get rid of when teaching is conducted virtually since most of the assessments online are in the input-output mechanism.

Hence, in this study, the researchers conducted an OLS focusing on radian measure to determine the possible didactical phenomena and problems that could arise when radian measure is taught via online synchronous class discussion.

Methods

To describe and narrate the experiences of the teachers and students in an online class, the researchers conducted an OLS which was focused on determining appropriate activities for an online class with consideration of the possible problems or dangers that didactical phenomena may bring alongside these activities. The students subject to this OLS were 24 first-year college students whose ages are 18-24 years old and taking Bachelor of Secondary Education major in Mathematics at President Ramon Magsaysay State University – Iba Campus. The OLS consists of three (3) phases: planning and designing the lesson, research lesson, and post-teaching discussion/teacher's reflection.

Planning and Designing the Lesson

The lesson plan format utilized was 4A's for Adult Learning that is composed of (a) Activities, (b) Analysis, (c) Abstraction, and (d) Application that are based on DepEd's ALS-EST Handbook for Implementers (2019), which is constructivist in nature and encourages active, independent, experiential, and cooperative learning.

The researchers collaboratively formulated the learning objectives based on the needs of the students and their compliance with the university's syllabus. Hence, the objectives for the lesson are: (1) define radian; (2) convert angle measure in degrees to radian and vice versa; and (3) solve problems involving radian. At this phase, the researchers had several meetings wherein a priori analysis was conducted on several possible teaching-learning situations. It helped them, especially the facilitator, identify the possible didactical phenomena that might occur within the time of implementation. Thus, strategies were critically predetermined to

address these things and ensure that the learning objectives would be fully achieved.

Research Lesson

Since the class would be done online, the researchers decided to use the Zoom Application as the teaching and learning platform. The facilitator who was in charge of implementing the chosen research lesson is one of the researchers and the mathematics instructor of the class as well. As mentioned, the utilization of 4A's lesson plan allowed the learners to do the following for each part of it:

- a. *Activities* – The facilitator started the class by manually grouping the students through the breakout Feature of the Zoom Application and instructed them to prepare any circular object and a measuring tool. Subsequently, he gave the task where they needed to measure the circumference and diameter of the object they had found in their home and let them complete the table provided. Each group had to accomplish the activity in ten (10) minutes.
- b. *Analysis* – Each member of every group initially performed the task individually. Afterward, the leader of each group was instructed to consolidate the observations of each member from the recently concluded activity, then present them to the class.
- c. *Abstraction* – The facilitator used these consolidated observations that each group presented as he introduced their topic for that day. Consequently, the facilitator started the discussion proper on the topic and used the ideas the learners shared during the activity.
- d. *Application* – At this point, the facilitator presented several examples that helped the students understand the concept of a radian and how it could be applied in real life. He also used an online assessment application (Quizizz) to make the discussion more engaging.

Post – Teaching Discussion/Teacher's Reflection

After the online class, the researchers started to have a post-discussion where a *posteriori* analysis took place. At the outset, the

chosen facilitator who implemented the research shared his experiences during the implementation of the OLS followed by the individual observations and recommendations of the other researchers/observers. They also assessed if they were able to achieve the lesson objectives set beforehand to the class and correspondingly if the purpose of the conduct of this OLS was attained. This phase of the OLS lasted for more than an hour.

Results and Discussions

The dialogues that took place during the post-lesson discussion were transcribed and analyzed by the researchers. Through this process, the researchers were able to observe three emerging themes central to the teaching of radian measure in an online set-up: (1) logistical issues in online teaching; (2) alignment of purpose and approach in online teaching; and (3) theoretical underpinnings for online teaching.

Logistical Issues in Online Teaching

The researchers selected the Zoom application as a point of convergence for the online class by considering its feature to have breakout rooms for the group activities. The teacher started the class by presenting the activity, explaining its mechanics, assigning the leaders for each group, and grouping the students through the breakout rooms. It could be observed that assigning students to the breakout rooms took some time. The observers and even the teacher identified some challenges that happened during the online teaching.

Online learning platforms pose limitations on teacher's supervision. Although the Zoom application is convenient for conducting group activities through its breakout rooms, it has a limit in providing the teacher the means of supervising all the groups simultaneously. The teacher can enter only one room at a time. The teacher handled this limitation by entering each room alternately from time to time. However, while he was present in one group, he was somewhat blind to what was happening in the other groups.

In all groups, it is notable that students had misconceptions about radius, ratio, and diameter. A student from Group 3 asked his groupmates, *"Is ratio half of the diameter? Is it correct?"* Another student from Group 2 expressed her confusion between radius and ratio when she asked the leader, *"Ate, we didn't get the radius, was it ratio?"* This is also evident in the discussions that occurred in Group 1:

Leader: *Are you done, Student 1?*

Student 1: *Not yet. How can I get the ratio?*

Leader: *What I know is... just divide the circumference by the diameter. Correct me if I'm wrong ha...*

Student 2: *Ate, how about the diameter?*

Leader: *It's the measure here.* (She was showing a paper with a circle on it). *For it to be easier for you, you can draw it on paper. Then, you just measure this distance.* (She was pointing to the diameter of the circle on her paper).

With the limited presence of the teacher in the breakout rooms, the students' willingness to cooperate with one another for the activity and their leaders' enthusiasm played an essential role in how they accomplished the activity. One of the observers commented, *"They have a problem in defining diameter, ratio, and radius. But I saw in their group during the breakout that they all were willing to cooperate. I have seen their willingness to really learn and contribute to the discussion. When their groupmates rectified the concepts during discussions, they got it immediately."* Two groups were able to present correct observations in the values for the ratio of the circumference and diameter of the circular objects they chose. Meanwhile, one group did not give a correct observation about the ratio of the circumference and diameter of the circular object. The ratio should be closed with the value of pi, but their answers are far from that value. This could be attributed to the difficulty that they have in measuring. As one observer shared, *"The computation of pi depends on the measurement. If the circumference and diameter measurements are inaccurate, their answers will be far from the value of pi."* Because of inaccuracy in the measurements of Group 2 for circumference and diameter, they could not observe that the ratio is close to the value of pi. Because of inaccuracy in the measurements

of Group 2 for circumference and diameter, they were not able to observe that the ratio is close to the value of pi.

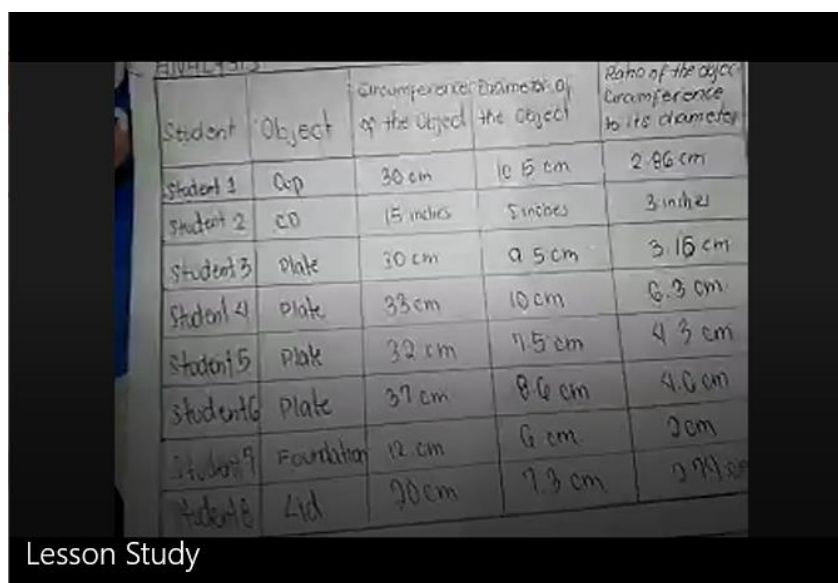
They even misunderstood the instruction for the observations. Instead of focusing their observations on the mathematical aspects of the activity, they considered their affective experiences while doing the activity. Their leader said, "Let's proceed with the observation. How do you feel? Did you have any difficulties in doing the activity?" Coincidentally the teacher entered their breakout room and clarified, "What you need to observe are the values of the ratio of your chosen objects' circumference and diameter."

During face-to-face learning, the misunderstandings and misconceptions of each group can be easily addressed by the teacher. On the other hand, for online learning, he needs assistant teachers for each group who will act as his extensions to overcome the limitation in supervising (in solving) all the breakout rooms at the same time.

Consequently, in conducting a lesson study in the normal face-to-face classroom, the observers are just there to track how the teacher implements the lesson, how the lesson unfolds, and how the students understand the lesson. It is suggested that in an online lesson study, the observers will take the role of assistant teachers in the absence of the teacher. As the need arises, they can give instructions and guidance

to the students. They can also process the ideas of the students. Moreover, the teacher can also have someone who will be the host of the Zoom meeting. This person will manage the admission of the students in the meeting room and breakout room. Hence, in an online lesson study, the teacher needs more assistance in managing learning activities.

Technological know-how affects the teacher's decisions. Another vital feature of the Zoom application is the multiple sharing of the screen that can be utilized to present the students' work for the group activity. However, this feature was not used in this online lesson study. When one student asked the teacher if he could share the screen, the teacher answered no. During the post-lesson discussion, the teacher was asked about this. He explained, "In our previous meeting, I also considered if they could share the screen for the presentation of their work. I ask the leaders if they have a laptop to teach them how to use the share screen. They said that they don't have a stable internet connection. So, I told them that they could just write it on cartolina or any paper. I assumed that they would not use the share screen. That is the reason why I reacted that way." The decision made by the teacher affected the presentation of the students. There were times that their outputs were not clearly visible on the screen.



Student	Object	Circumference of the Object	Diameter of the Object	Ratio of the object's Circumference to its diameter
Student 1	Cup	30 cm	10.5 cm	2.86 cm
Student 2	CD	15 inches	5 inches	3 inches
Student 3	Plate	30 cm	9.5 cm	3.16 cm
Student 4	Plate	33 cm	10 cm	3.3 cm
Student 5	Plate	32 cm	7.5 cm	4.3 cm
Student 6	Plate	37 cm	8.6 cm	4.0 cm
Student 7	Fountain	12 cm	6 cm	2 cm
Student 8	Lid	20 cm	7.3 cm	2.74 cm

Figure 1. The output presented by Group 2

Although the teacher is aware of the Zoom application feature for multiple sharing of the screen, his decision affected how Zoom can be maximized for the furtherance of the students' learning experience for the lesson about radian measure. This result pointed out that knowing how to navigate the online learning applications affects the teacher's decisions during planning and even implementing the lesson. Having enough knowledge about the features of the online learning applications can help teachers make mindful decisions to help students learn better during the actual online lesson.

Alignment of Purpose and Approach in Online Teaching

The researchers designed the research lesson to provide a learning experience that will allow students to conceptualize radian measure even through a distance learning delivery modality. Their aim was not just for the students to know how to convert an angle measure from degrees to radians and vice versa but also for the students to understand the concept of radian measure.

It was evident in the online lesson that the teacher presented the concept behind the algorithm of how the formula was derived for the conversion of angle measures. However, during the post-lesson discussion, the following suggestions emerged for the teacher to strengthen the students' conceptual understanding of radian measure.

- The historical perspective of radian measure can be included in the discussion for the students to know the importance of radian measure and when it is appropriate to use compared to degrees.
- There is need for clarification if radian is a unit. It should have been explained to the students whether it is crucial to write the word rad or not after the degree measure in radian.
- Students should be allowed to solve on their own. The teacher should have been shown the solutions for the examples after the students tried to solve them.
- The problem given as assessment can be revised so that the students will appreciate more the application of radian measure in real-life.

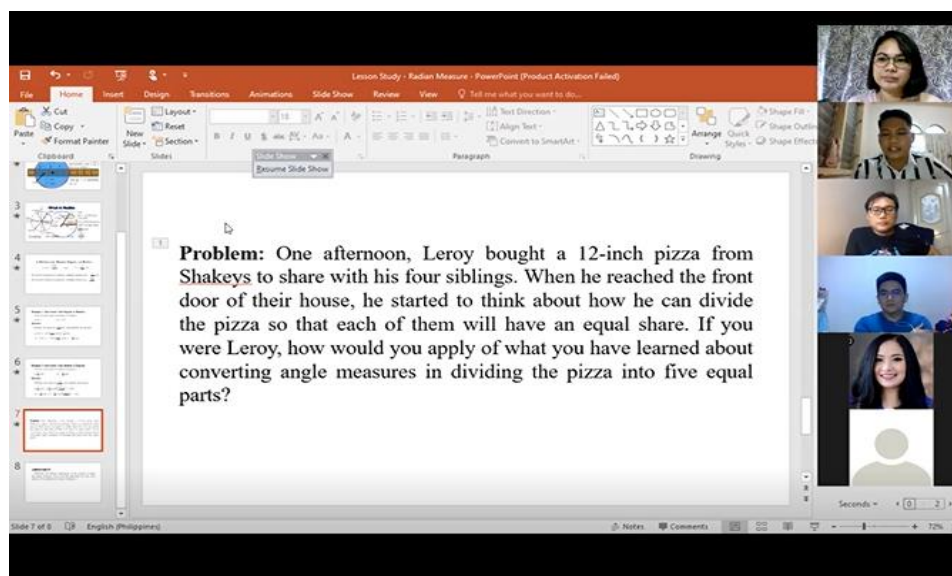


Figure 2. The word problem in the assessment

One of the observers shared, *"I realized after discussing this problem that yes, radian measure is applicable in solving this problem. However, I thought, what if we applied the concept to something practical that the students can*

use after learning about it. Some students solved the word problem about the pizza by calculating its circumference using the given diameter and dividing the resulting circumference by five. But at the end of that, can they really use the process

that they did in solving problems in real life? We need to think of a new lesson application relevant to students' day-to-day lives."

Another observer added, "When you divide the pizza, you will look at the center. From the center, you can divide it into five parts. In a situation like distance traveled, if you are from the center, that's degrees, but if your point of view is on the outer part, what will be traveled, that's radians. Through the context of distance traveled by a circular object like the wheels of a car, maybe the students will see the importance of using radians to solve the problem."

All activities included in the lesson and every teacher's undertaking in implementing the lesson must always be aligned with the lesson's purpose which is to develop students' understanding about the concept of radian and its application in real-life. This could be done by being mindful of designing the lesson's learning activities and being assertive during the actual teaching.

Theoretical Underpinnings for Online Teaching. With the different challenges linked to online learning, the researchers used the Multiple Instructional Principles and Didactical Phenomena, as guiding theories in planning the research lesson.

Multimedia Instructional Principles. To adhere to extraneous processing, the researchers agreed that the topic about degree measure

would be taught separately, and it will be done prior to teaching radian measure. Through this, the students could focus entirely on learning radian measures during the online lesson study. However, during the post-lesson discussion, the researchers realized the removal of some parts of the lesson which do not target the instructional goal. Following the coherence principle, the preliminary portion of the activity could be deleted from the actual online lesson. The students took 30 minutes to finish the activity. They used a large amount of time to measure the circumference and diameter and compute the ratio instead of allotting it to discuss their observations. This result pointed out that the activity could be structured in such a way that the time for the online class can be maximized for the discussion of the students about their observations.

One of the observers suggested, "The measuring and computation of ratio can be given as a preliminary task to the lesson. In a way, students will have time to search the definition of circumference, diameter, and ratio and how to measure and compute them." By assigning the preliminary task before the online class, the students could unlearn their misconceptions independently. Accordingly, they will have more time for observing and comparing the results of their measurements and computation. This will also pave the way for a deeper discussion among the students.

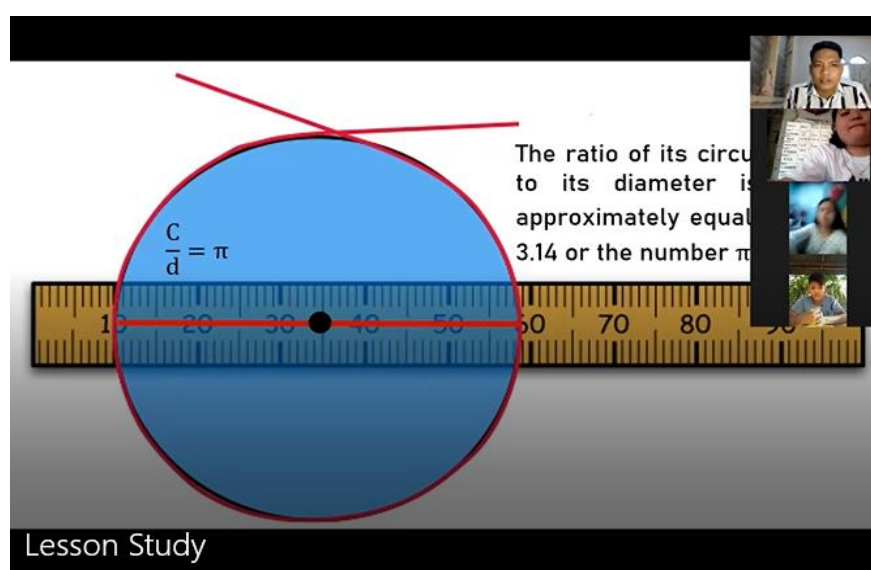


Figure 3. The teacher explains the activity that the student did through an illustration

The researchers also considered necessary processing to design the activity to help students know about the key concepts relevant to radian measure. Generative processing was also evident in the implementation of the research lesson. The teacher used an illustration (see Figure 3) to explain what they did in the activity. Instead of just using his words, the teacher opted to use both words and illustration. This action made by the teacher follows the multimedia principle.

Black Box Effect. Out of the different didactical phenomena, this has been the major

consideration of the researchers while they were looking for an online application for the exercises of the students. They searched for an online application that would tell the students if their answer was correct and provide a discourse on why their answer was right or wrong. The teacher found Quizizz, which has a feature that shows the solution for the correct answer. However, this feature can only be accessed upon subscription. So, it was utilized by the teacher, excluding that feature.



Figure 4. The online application for the exercises of the students

Although some students could not participate in answering the exercises due to unstable internet connections in their area, most of the students enjoyed answering the exercises through this application. The researchers' professor commented, "This is (Quizizz) very good, but we cannot see the discourse upon whether the answer is right or wrong. It could be a good springboard for starting a discourse around a wrong answer." The researchers could have avoided the Black Box effect when they planned how to utilize Quizizz so that discourse could be opened for the students. After answering the online exercises, the teacher can share the solution of each item and let the students observe and identify what made their answer their solution and answer wrong. Even with the limited

features of an online application, it is still possible to avoid the Black Box effect and the other didactical phenomena through a well-planned lesson.

Conclusion

This study showed that an LS conducted under a virtual environment where all the principal characters involved participate via the online modality is possible.

First, it cannot be denied that one of the factors that would make this form of LS a success depends largely on the strength and consistency of internet connectivity of the participants at the time of the session. On the other hand, regardless of the modality, just like in any LS, preparation on the part of both the teacher

and the learners is the key. To be more specific, the teachers/facilitators should start the preparation processes by ensuring that the prerequisite mathematical concepts to the present topic, in this case, Radian, are fully understood by the learners to ensure the constructive alignment of the teaching and learning processes. Further, such preparations should have also included contingency measures that would address incidents like participants being disconnected in the middle of the OLS or unable to analyze situations requiring prior mathematical knowledge. Another issue related to preparation is the conduct of the activities embedded in the lesson proper. Some activities that can be done ahead of time (*for this specific session, the activity on measuring and calculating the ratio of the circumference to the diameter of a circle*) should be predetermined and implemented beforehand in order to maximize the actual teaching time and focus on achieving the set objectives. Nevertheless, despite some issues that stalled the students from reaching the desired goals (*deriving π and consequently understanding radians*), their active participation in the activity was truly noteworthy. Most of all, the lesson objectives were successfully met, as proven by the majority of the students' success in solving and answering the subsequent quizzes.

All things considered, it can be inferred from this OLS that teaching mathematics subject per se could be a challenging task regardless of teaching modality and platform. However, by ensuring a complete, well-defined, and concrete plan, it can be guaranteed that every aspect of the teaching-learning process will be implemented in the best way possible. In this regard, conducting a similar research paper would greatly help further establish and develop the current method of online mathematics teaching.

Acknowledgment

The researchers would like to express their warmest gratitude and sincerest appreciation to the BSEd I – Mathematics students at President Ramon Magsaysay State University - Iba Campus under the College of Teacher Education, who served as the participants of the study.

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